Image Characterization by Morphological Hierarchical Representations

PhD Defense

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Plan

Plan

Introduction

Stochastic Watershed (SWS) Hierarchies

Introducing prior spatial information

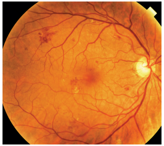
Combinations of hierarchies

Structuring the space of hierarchies

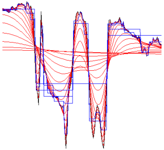
Conclusion

Information is intrinsically multi-scale

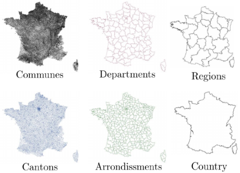
(a)



(b)



(c)



(d)

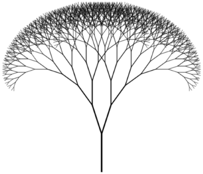
- Single-scale observation is very restrictive.
- Need for **multi-scale representations**.

Multi-scale representations

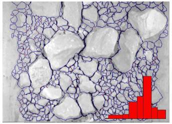
- Decomposing images into fundamental elements easily interpreted
- Linked with human perception

- Scale-space theory [1]
- Fractal model
- Granulometry
- Hierarchical partitioning

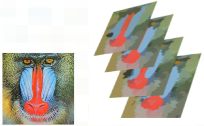
(a) Levels of a scale-space decomposition.



(b) Fractal model



(c) Granulometry approach



(d) Hierarchical partitioning

[1] Lindeberg, T., and Bart M.H.R. "Linear scale-space I: Basic theory." Geometry-Driven Diffusion in Computer Vision. Springer, 1994. 1-38.

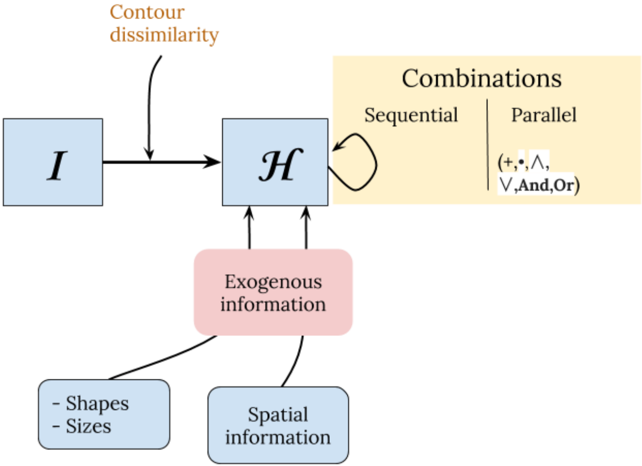
A variety of morphological hierarchical models

In the literature:

- Tree-of-shapes
- Max-tree, min-tree
- Constrained connectivity

In the thesis:

- Watershed hierarchies
- Waterfall hierarchy
- Binary-scale climbing hierarchy
- Hierarchies of levelings
- **Stochastic Watershed (SWS) hierarchies**



Multiplying the viewpoints

A single scale is usually not sufficient

- The information is often distributed across scales.

→ **Hierarchical representation**

A single hierarchy is usually not sufficient

- There is no single hierarchy that captures all the desired features.

→ **Multi-model** approach by considering **several hierarchies**

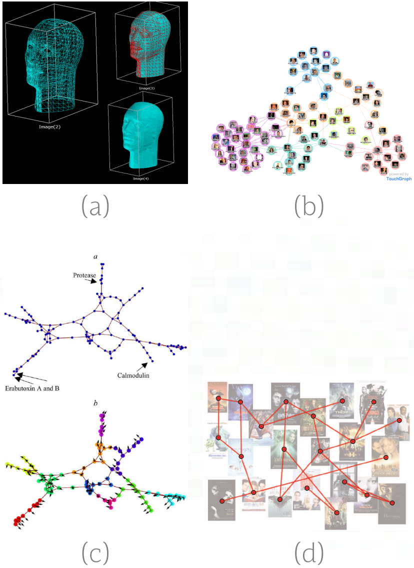
Thesis goal:

Multiply and combine morphological hierarchical representations to be used in various applications

Graph-based framework

Choice to work with **graphs**:

- Independent from dimension
- Powerful tools available
- Can represent various objects (images, social networks, etc.)



(a) (b) (c) (d)

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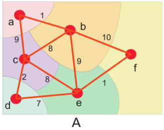
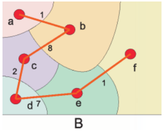
Hierarchies on graphs

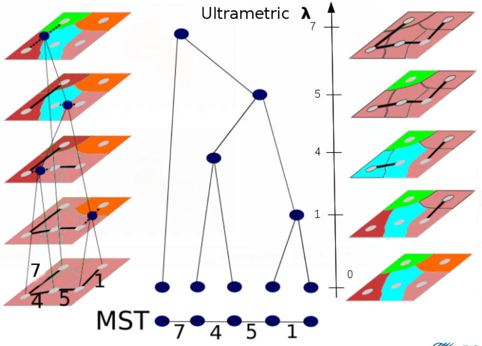


(a) Region Adjacency Graph



(b) Minimum Spanning Tree

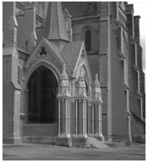
- Cutting edges of the MST by decreasing valuations → progressive fusion of regions
  - Minimum Spanning Forest (MSF) hierarchy $(\mathcal{H}, \boldsymbol{\lambda})$
  - $\boldsymbol{\lambda}$: **ultrametric distance**
  - Can be modeled as a tree called **dendrogram**
  - Can be visualized as a **saliency map**



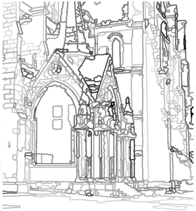
(a)

Hierarchies on graphs

- (Zahn)[2]Inconsistent edges are cut first
- **Trivial** hierarchy: edges weighed according to **local** cues (gradient)
  - Myopic
  - Chaining effect
- Need to enlarge the information support



(a) Image



(b) Trivial hierarchy saliency map

→ Need to redefine edge weights to highlight significant contours

[2] Zahn, C. T. (1971). Graph-theoretical methods for detecting and describing gestalt clusters. IEEE Transactions on computers, 100(1), 68-86.

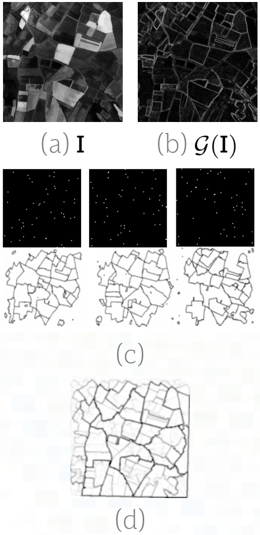
Stochastic Watershed [3]

Process

- Iterate N times:
  - Draw random markers $\mathbf{m}_i$
  - Compute associated watershed $W_{\mathbf{m}_i}(\mathbf{I})$
- $\tilde{W}(\mathbf{I}) = \frac{\sum_{i \in \{1,\dots,N\}} W_{\mathbf{m}_i}(\mathbf{I})}{N}$

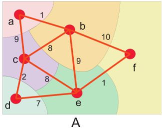
Non-local estimation of contours strength

- Computationally heavy
- Fuzzy contours

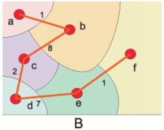


[3] Angulo, J., & Jeulin, D. (2007, October). Stochastic watershed segmentation. In PROC. of the 8th International Symposium on Mathematical Morphology (pp. 265-276).

Stochastic Watershed on Graphs



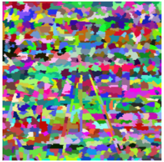
(a) Region Adjacency Graph



(b) Minimum Spanning Tree

Graph built upon a **fine partition** of the image[4]:

(c) Image



(d) Waterpixels

(e) Mosaic image

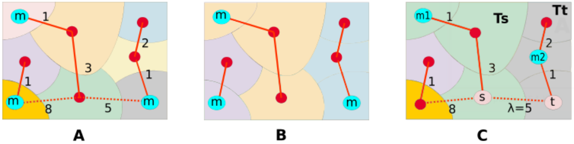
[4] Machairas, V., Faessel, M., Cárdenas-Peña, D., Chabardes, T., Walter, T. and Decencière, E., 2015. Waterpixels. IEEE Transactions on Image Processing, 24(11), pp.3707-3716.

Stochastic Watershed on Graphs

Concept

Idea

- **Input**: any hierarchy on a MST
- Markers: **random sampling**
- **Output**: new MST valuations



Marker-based segmentation

For an edge e_{st} of the MST:

- If $\eta_{st} = \lambda$, we cut edges with weights $> \lambda$.
- Subtrees T_s and T_t underlie regions R_s and R_t.
- e_{st} is a segmentation frontier iff:
  - $\exists$ at least one marker in R_s
  - $\exists$ at least one marker in R_t

Stochastic Watershed on Graphs

Calculus

For any markers distribution:

$$\begin{aligned}\theta_{st} &= \Pr(\{\exists \text{ at least one marker in } R_s\} \text{ AND} \{\exists \text{ at least one marker in } R_t\}) \\ &= 1 - \Pr(\{\nexists \text{ marker in } R_s\} \text{ OR} \{\nexists \text{ marker in } R_t\}) \\ &= 1 - \Pr(\{\nexists \text{ marker in } R_s\}) - \Pr(\{\nexists \text{ marker in } R_t\}) + \Pr(\{\nexists \text{ marker in } R_s \cup R_t\})\end{aligned}$$

For a distribution of markers following a **uniform Poisson process**:

- $\Pr(\{\exists \text{ marker in } R\}) = 1 - \exp^{-\frac{a}{S}\omega}$, when drawing ω markers.

$$\begin{aligned}\Rightarrow \theta_{st} &= 1 - \Pr(\{\nexists \text{ marker in } R_s\}) - \Pr(\{\nexists \text{ marker in } R_t\}) + \Pr(\{\nexists \text{ marker in } R_s \cup R_t\}) \\ &= 1 - \exp^{-\frac{a_s}{S}\omega} - \exp^{-\frac{a_t}{S}\omega} + \exp^{-\frac{(a_s+a_t)}{S}\omega}\end{aligned}$$

Stochastic Watershed on Graphs

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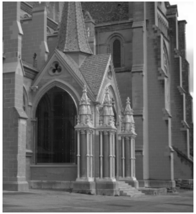
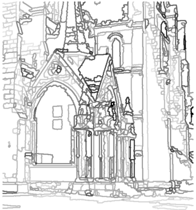
$$= 1 - \exp^{-\frac{a_s \times \lambda_{st}}{S \times \lambda_{max}}\omega} - \exp^{-\frac{a_t \times \lambda_{st}}{S \times \lambda_{max}}\omega} + \exp^{-\frac{(a_s + a_t) \times \lambda_{st}}{S \times \lambda_{max}}\omega}$$

Stochastic Watershed on Graphs

Illustration



(a) Image



(b) Trivial hierarchy

(c) Area-based SWS hierarchy

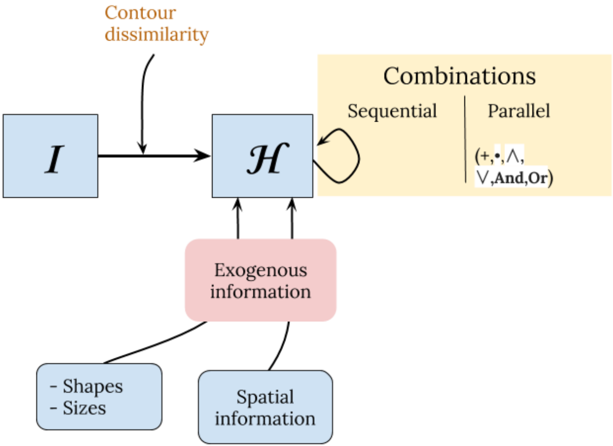
(d) Volume-based SWS hierarchy

Stochastic Watershed on Graphs

A great versatility

We can act on:

- **Initial dissimilarity**
- **Type of SWS hierarchy**: area-based, volume-based, symmetrical or not
- **Markers**: points or sets, regionalized or not
- **Probability laws** governing markers distributions: homogenous, non-homogenous, a priori or learned



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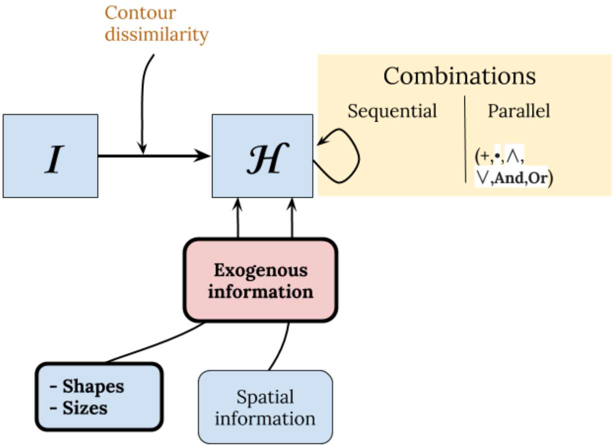

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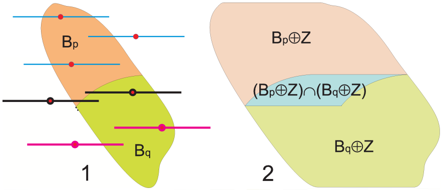
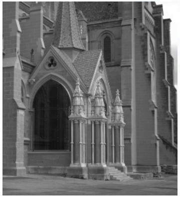
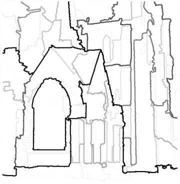


(a)



(b) Image



(c) Vertical markers

(d) Horizontal markers

Plan

Stochastic Watershed on Graphs

A great versatility

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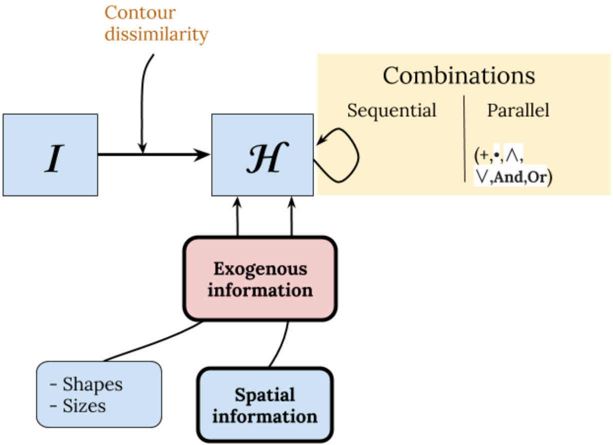
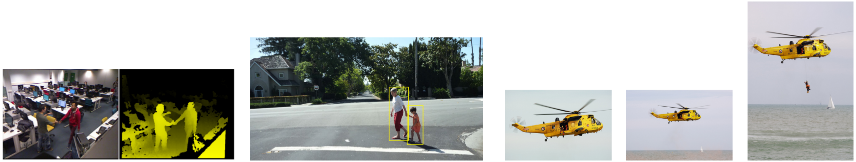
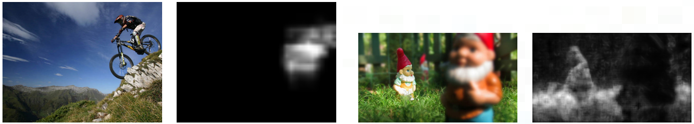


A growing number of spatial information sources



(a) (b) (c)



(d) (e)

- Problem-specific spatial information
- Multimodal images

How can we use them to **pilot the hierarchical segmentation process**?

SWS model adaptation

Markers spread following a Poisson process

For a region R:

- $\Lambda(R)$ = mean value of the number of markers falling in R
- $\Pr(\nexists$ marker in $R) = \exp^{-\Lambda(R)}$

Choice of density

- Homogenous density λ: $\Lambda(R) = \text{area}(R)\lambda$,
- Non-uniform density λ: $\Lambda(R) = \int_{(x,y)\in R} \lambda(x,y)\,\mathrm{d}x\mathrm{d}y$

Hierarchy with Regionalized Fineness (HRF)

Exogenous information

- E: object or class of interest
- θ_E: probability density function (PDF) associated with E on the domain D of the image I
- $\mathrm{PM}(\mathbf{I}, \theta_E)$: probabilistic map associated, in which each pixel $p(x, y)$ of $\mathbf{I}$ takes as value $\theta_E(x, y)$ its probability to be part of E

New e_{st} valuation:

$$\theta_{st} = 1 - \exp^{-\Lambda(\mathrm{R}_s)} - \exp^{-\Lambda(\mathrm{R}_t)} + \exp^{-\Lambda(\mathrm{R}_s \cup \mathrm{R}_t)}$$

$$\Lambda(\mathrm{R}) = \mathrm{area}(R)\lambda$$

Hierarchy with Regionalized Fineness (HRF)

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- $\mathrm{PM}(\mathbf{I}, \theta_E)$: probabilistic map associated, in which each pixel $p(x, y)$ of $\mathbf{I}$ takes as value $\theta_E(x, y)$ its probability to be part of E

Key idea

$$\theta_{st} = 1 - \exp^{-\Lambda_E(\mathrm{R}_s)} - \exp^{-\Lambda_E(\mathrm{R}_t)} + \exp^{-\Lambda_E(\mathrm{R}_s \cup \mathrm{R}_t)}$$

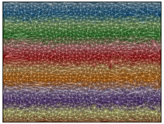
$$\Lambda_E(\mathrm{R}) = \int_{(x,y)\in\mathrm{R}} \theta_E(x, y)\lambda(x, y)\, \mathrm{d}x\mathrm{d}y$$

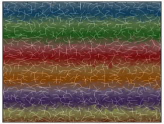
Methodology

- Compute the fine partition π_0, RAG $\mathcal{G}$, $\mathcal{MST}(\mathcal{G})$


(f) Image

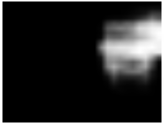

(g) Mosaic

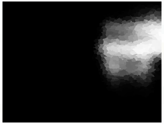

(h) RAG


(i) MST

- Compute a probabilistic map $\pi_\mu = \pi_\mu(\pi_0, \mathrm{PM}(\mathbf{I}, \theta_E))$


(j) Image


(k) Probability to be a bike


(l) π_μ

- compute new values of edges using previous formulas

Application 1: Scalable transmission favoring regions of interest

Prior: face detection [5]

(m) Image

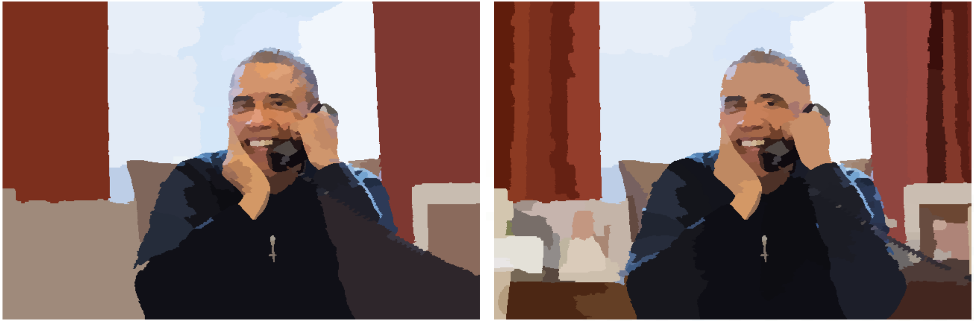
(n) Probability map associated with “Face” class

Figure : Face detection using Haar wavelets

[5] Viola, P., & Jones, M. (2001). Rapid object detection using a boosted cascade of simple features. In CVPR 2001. Proceedings of the 2001 IEEE CSC (Vol. 1, pp. I-I). IEEE.

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 200 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 175 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

(a) Non homogenous law

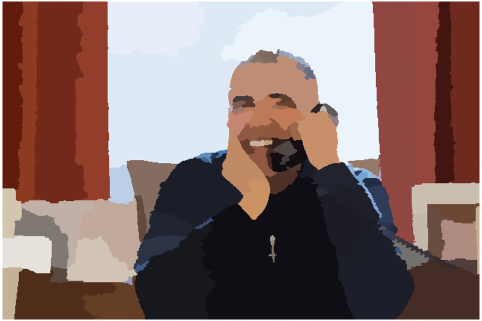
(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 150 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

(a) Non homogenous law



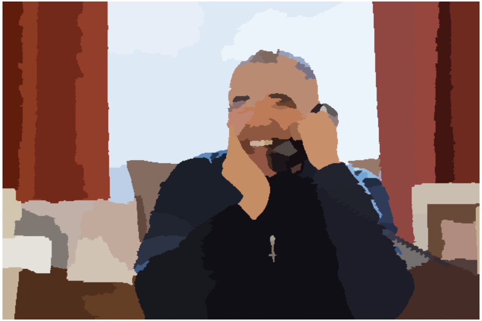
(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 125 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

(a) Non homogenous law



(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 100 regions

Application 1: Scalable transmission favoring regions of interest

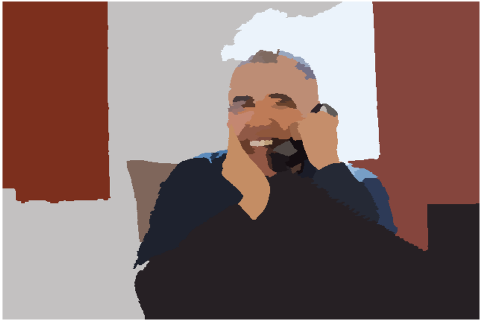
Volume-based SWS hierarchy

(a) Non homogenous law (b) Homogenous law

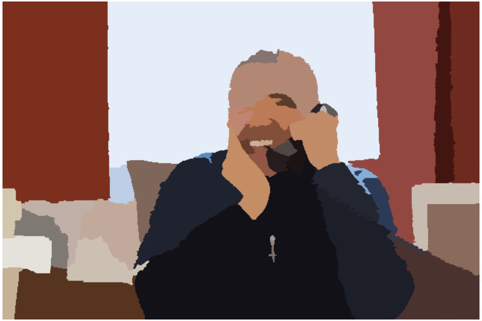
Figure : Comparison between HRF and hierarchy with homogenous law - 75 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law



(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 50 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 25 regions

Application 1: Scalable transmission favoring regions of interest

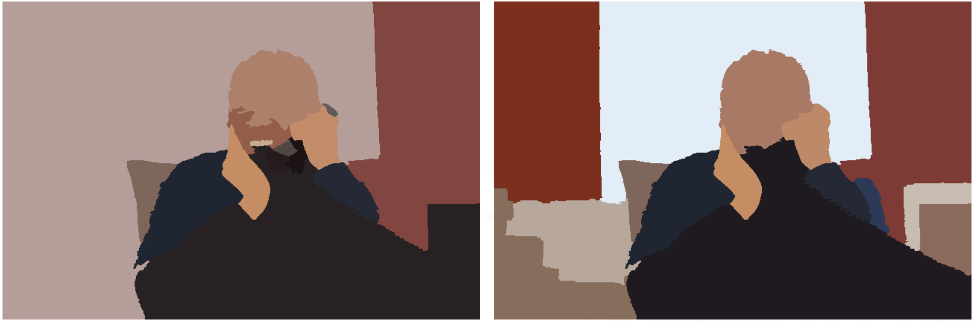
Volume-based SWS hierarchy

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 20 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 15 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 10 regions

Application 1: Scalable transmission favoring regions of interest

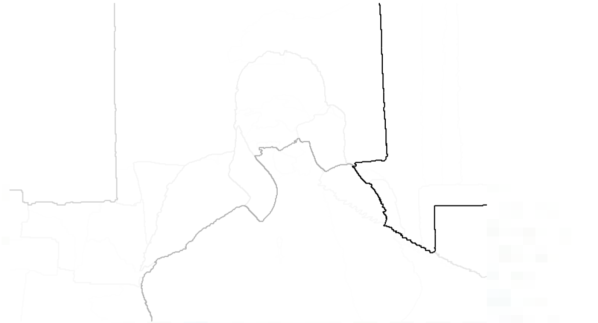
Volume-based SWS hierarchy

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 5 regions

Application 1: Scalable transmission favoring regions of interest

(a) Non homogenous law



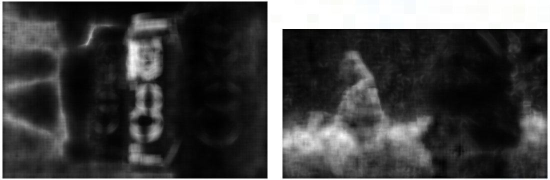
(b) Homogenous law

Figure : Saliency images

Application 1: Scalable transmission favoring regions of interest

Prior: non-blur zones detection [6]

(a) Images

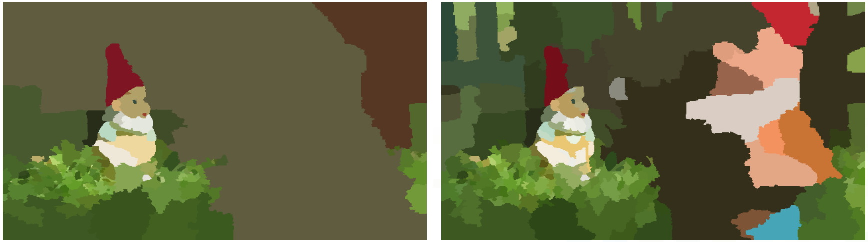


(b) Probability maps of non-blur zones

[6] Su, B., Lu, S., & Tan, C. L. (2011, November). Blurred image region detection and classification. In Proceedings of the 19th ACM international conference on Multimedia (pp. 1397-1400). ACM.

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

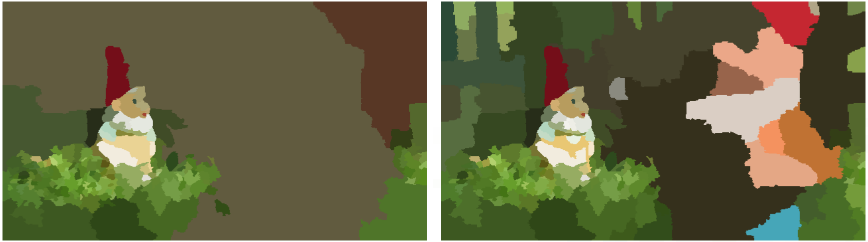


(c) Non homogenous law　　(d) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 200 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

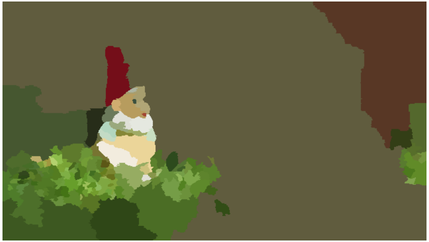


(a) Non homogenous law (b) Homogenous law

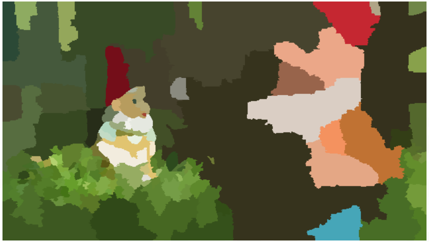
Figure : Comparison between HRF and hierarchy with homogenous law - 175 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law

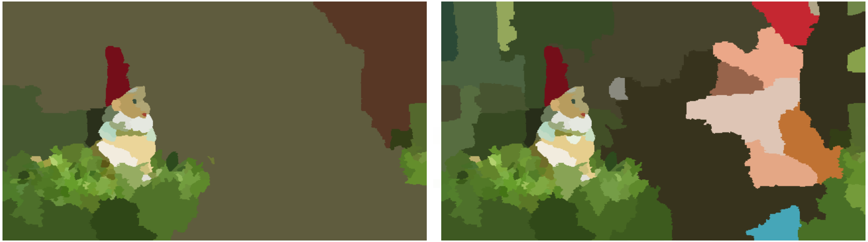


(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 150 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

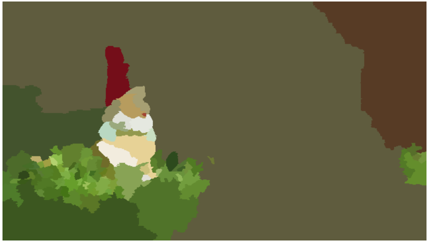


(a) Non homogenous law (b) Homogenous law

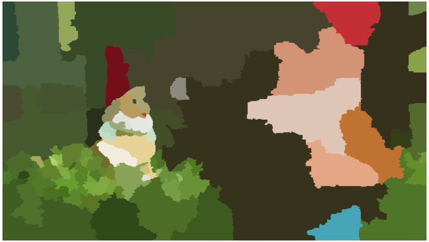
Figure : Comparison between HRF and hierarchy with homogenous law - 125 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law

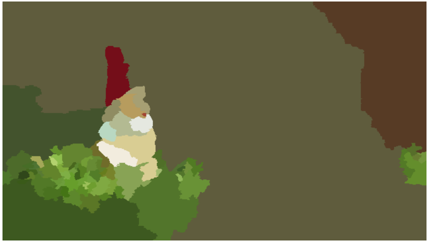


(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 100 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

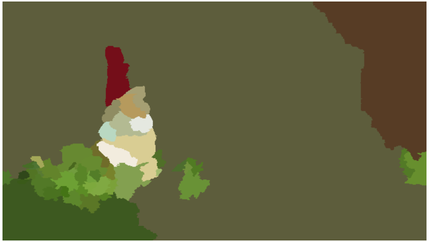

(a) Non homogenous law


(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 75 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



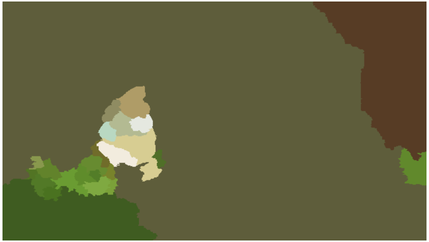
(a) Non homogenous law

(b) Homogenous law

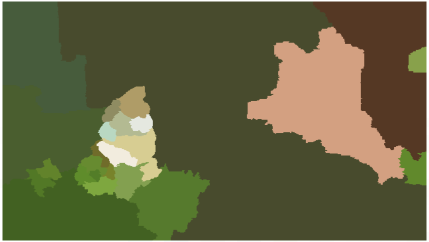
Figure : Comparison between HRF and hierarchy with homogenous law - 50 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law

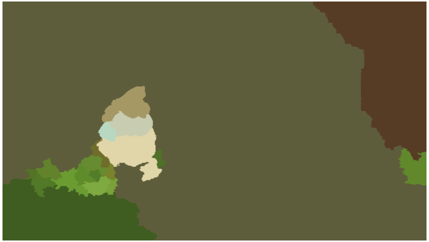


(b) Homogenous law

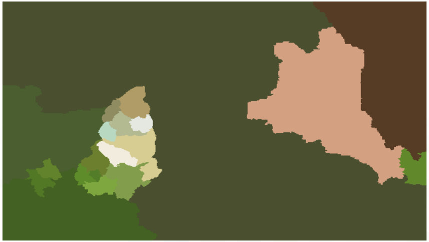
Figure : Comparison between HRF and hierarchy with homogenous law - 25 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law

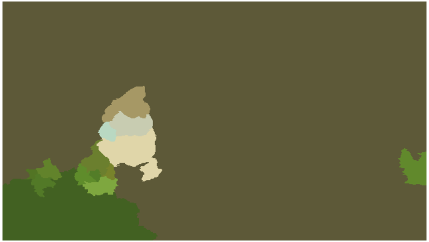


(b) Homogenous law

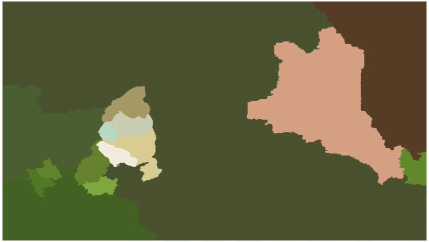
Figure : Comparison between HRF and hierarchy with homogenous law - 20 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



(a) Non homogenous law

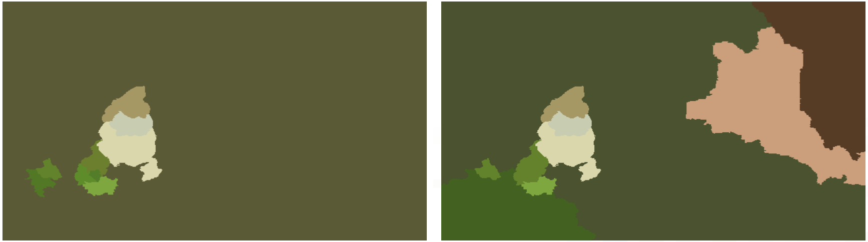


(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 15 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy



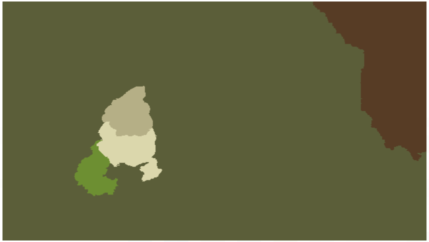
(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 10 regions

Application 1: Scalable transmission favoring regions of interest

Volume-based SWS hierarchy

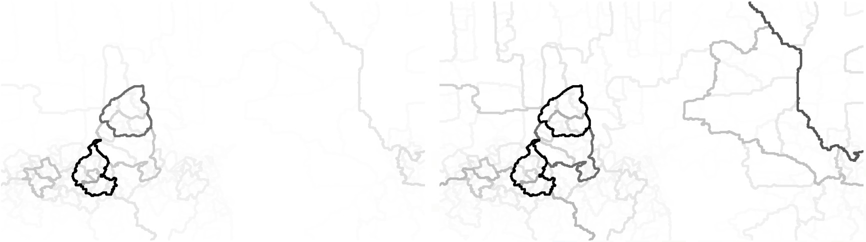
(a) Non homogenous law



(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 5 regions

Application 1: Scalable transmission favoring regions of interest



(a) Non homogenous law (b) Homogenous law

Figure : Saliency images

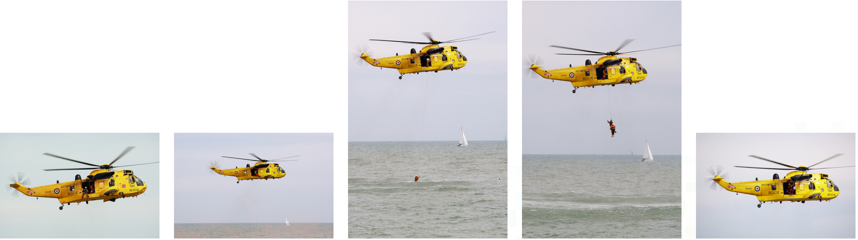
Application 1: Scalable transmission favoring regions of interest

(a) Non homogenous law (b) Homogenous law

Figure : Saliency images

Application 2: co-hierarchical segmentation

Images from iCoSeg database [7].

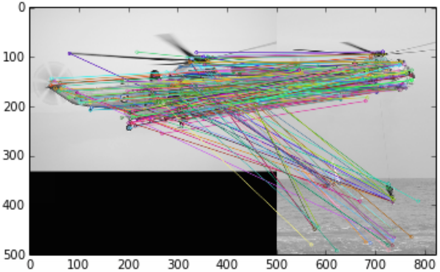


(a) (b) (c) (d) (e)

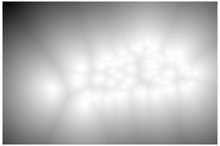
Figure : Images to co-segment

[7] Batra, Dhruv, et al. "Interactively co-segmentating topically related images with intelligent scribble guidance." International journal of computer vision 93.3 (2011): 273-292.

Application 2: co-hierarchical segmentation



(a) Prior



(b) Associated probability map

- Matching of interest points SIFT/SURF/ORB between the image to segment and all other images of the class.
- Retain all matched keypoints.

Application 2: co-hierarchical segmentation

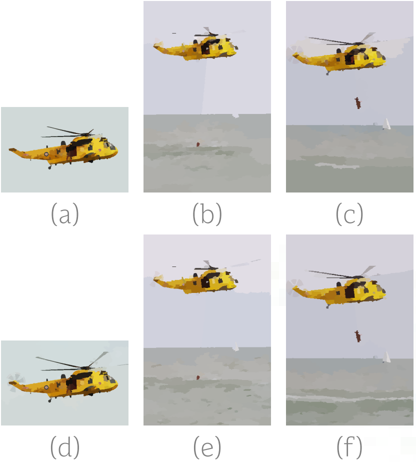


Figure : Comparison between HRF and hierarchy with homogenous law - 200 regions

Application 2: co-hierarchical segmentation

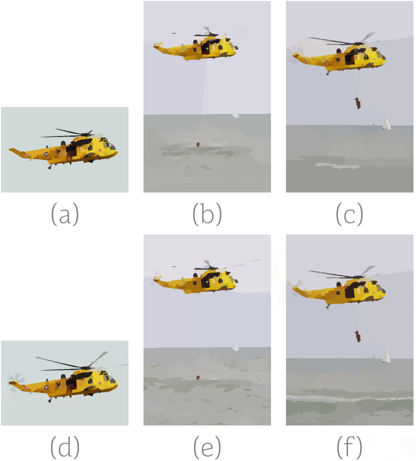


Figure : Comparison between HRF and hierarchy with homogenous law - 175 regions

Application 2: co-hierarchical segmentation

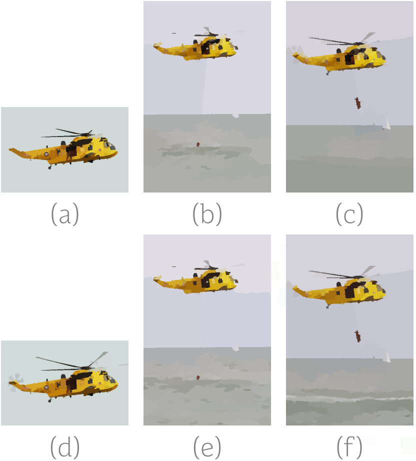


Figure : Comparison between HRF and hierarchy with homogenous law - 150 regions

Application 2: co-hierarchical segmentation

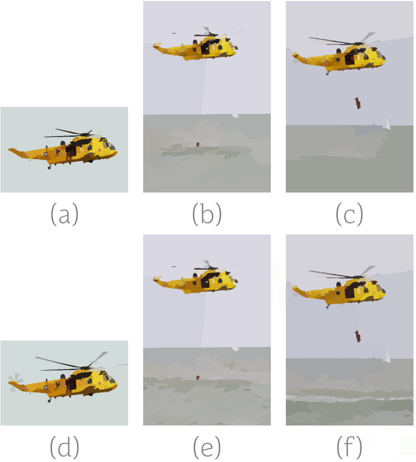


Figure : Comparison between HRF and hierarchy with homogenous law - 125 regions

Application 2: co-hierarchical segmentation

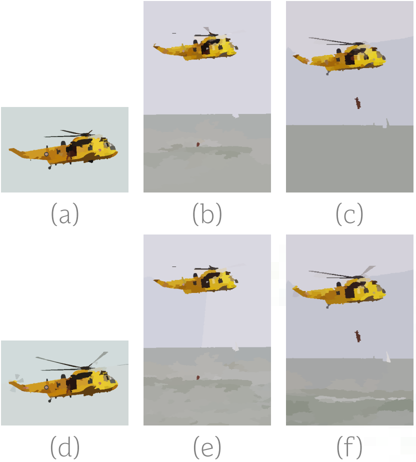


Figure : Comparison between HRF and hierarchy with homogenous law - 100 regions

Application 2: co-hierarchical segmentation

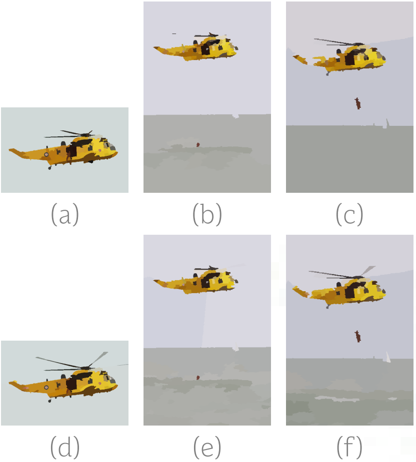


Figure : Comparison between HRF and hierarchy with homogenous law - 75 regions

Application 2: co-hierarchical segmentation

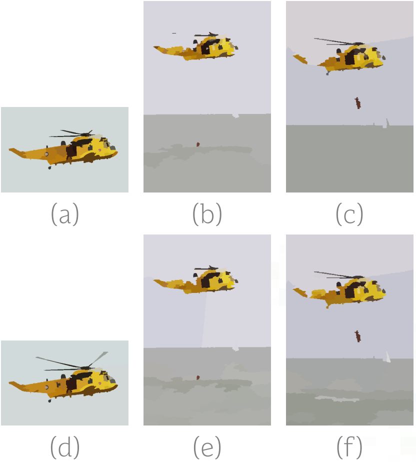


Figure : Comparison between HRF and hierarchy with homogenous law - 50 regions

Application 2: co-hierarchical segmentation

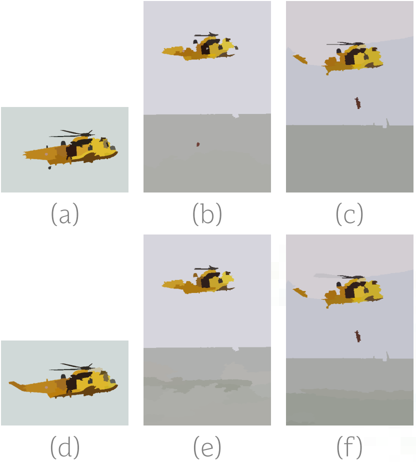


Figure : Comparison between HRF and hierarchy with homogenous law - 25 regions

Application 2: co-hierarchical segmentation

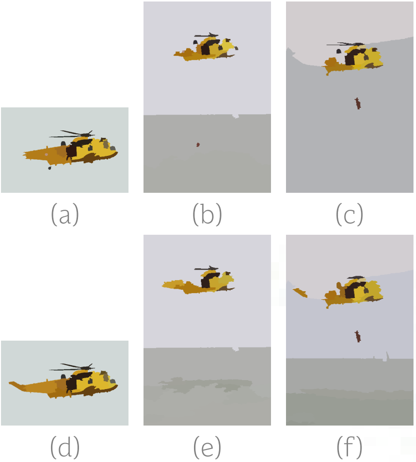


Figure : Comparison between HRF and hierarchy with homogenous law - 20 regions

Application 2: co-hierarchical segmentation

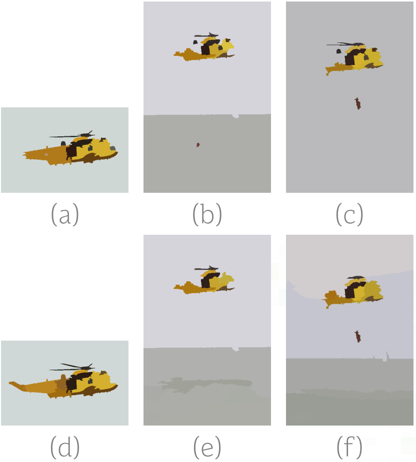


Figure : Comparison between HRF and hierarchy with homogenous law - 15 regions

Application 2: co-hierarchical segmentation

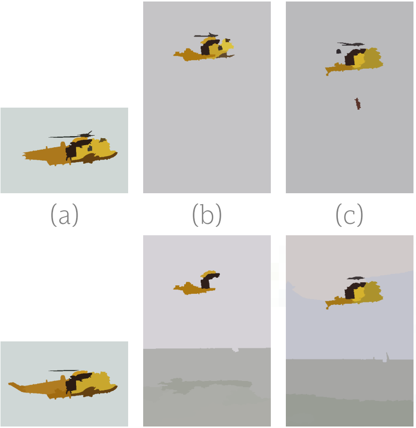

(a) (b) (c) (d) (e) (f)

Figure : Comparison between HRF and hierarchy with homogenous law - 10 regions

Application 2: co-hierarchical segmentation

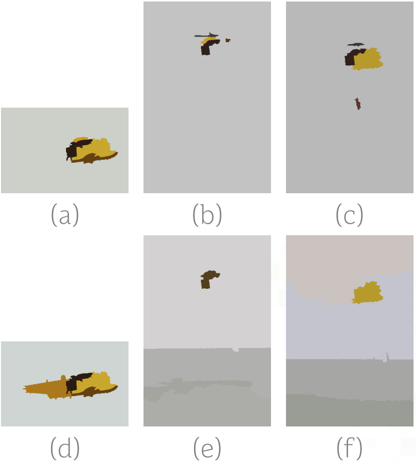


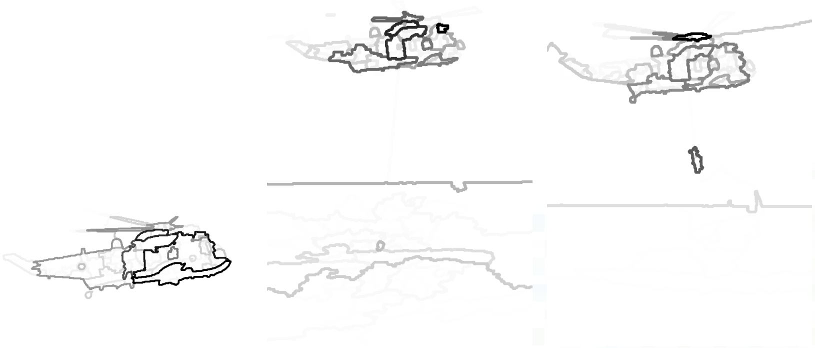
Figure : Comparison between HRF and hierarchy with homogenous law - 5 regions

Application 2: co-hierarchical segmentation

(a) (b) (c)

Figure : Saliency images for homogenous process

Application 2: co-hierarchical segmentation

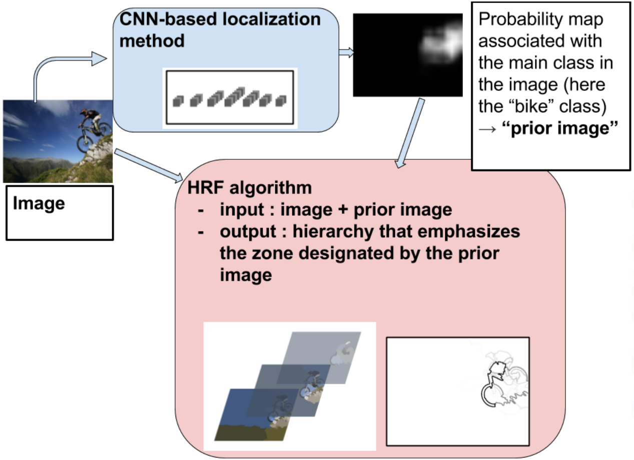


(a) (b) (c)

Figure : Saliency images for non-homogenous process

Application 3: weakly-supervised hierarchical segmentation

Weakly-supervised HRF algorithm



Application 3: weakly-supervised hierarchical segmentation

CNN-based localization method

VGG16 reference CNN classifier, trained on ImageNet [8].

- Input: image in 224×224 pixels
- Output = vector of size 1000, appearance probability of each class.

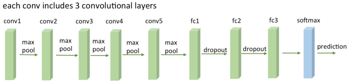
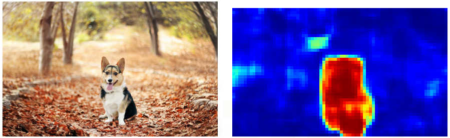


Figure : VGG16 Network Architecture [9]



(a) Image

(b) Heatmap output by CNN-based method

Figure : Generation of probability maps [10]

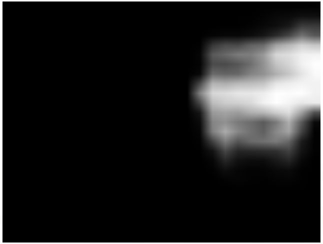
[8] http://image-net.org/

[9] http://www.robots.ox.ac.uk/vgg/research/very_deep/

[10] M. Oquab, L. Bottou, I. Laptev, J. Sivic; "Is Object Localization for Free? - Weakly-Supervised Learning With Convolutional Neural Networks", in CVPR, 2015, pp. 685-694

Application 3: weakly-supervised hierarchical segmentation

(a) Image



(b) Prior: main class localization

Figure : Image and localization image

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 95 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 90 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 85 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 80 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 75 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 70 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 65 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 60 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 55 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 50 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 45 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 40 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 35 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

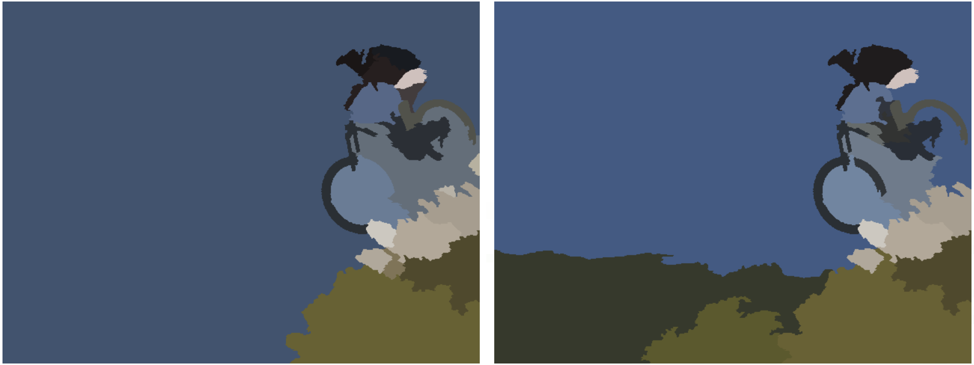
Figure : Comparison between HRF and hierarchy with homogenous law - 30 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 25 regions

Application 3: weakly-supervised hierarchical segmentation



(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 20 regions

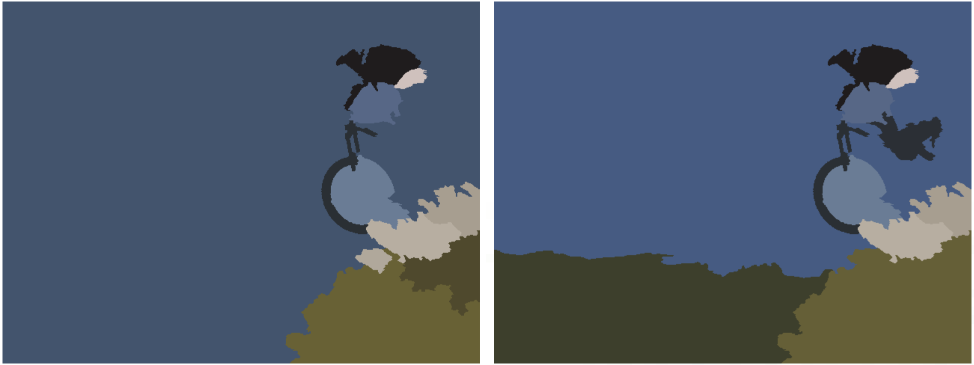
Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

(b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 15 regions

Application 3: weakly-supervised hierarchical segmentation



(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 10 regions

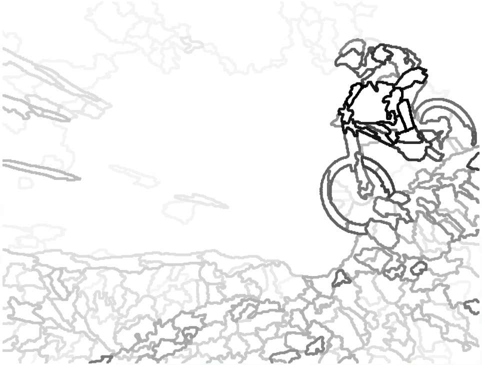
Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law (b) Homogenous law

Figure : Comparison between HRF and hierarchy with homogenous law - 5 regions

Application 3: weakly-supervised hierarchical segmentation

(a) Non homogenous law

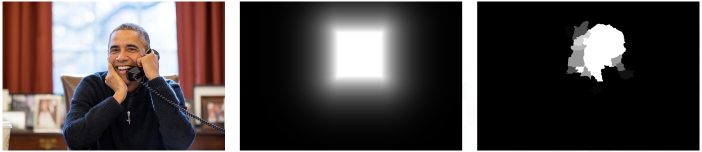


(b) Homogenous law

Figure : Saliency images

Perspectives

- Go further: have specific markers depending on the regions
- Use such hierarchies to refine the output of a segmentation module



(a) (b) (c)

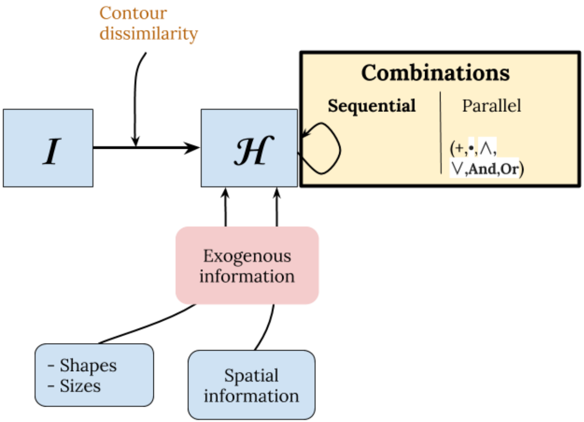
- Towards sequential refinings

Plan

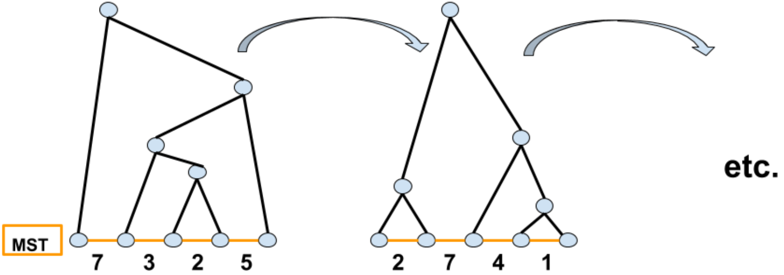
Combinations of hierarchies

Hierarchies can be combined to express complex properties

- **Sequential combinations** by chaining



Combinations of hierarchies
Sequential combinations by chaining



Computations made on the dendrogram lead to new MST's valuations.

A new structure emerges with these new valuations, after a reorganization of internal nodes.

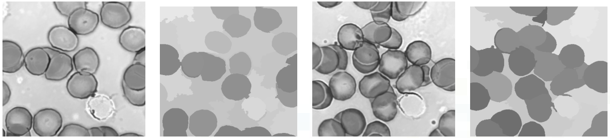
We can repeat the process with this new structure as departure point → **'chaining of hierarchies'**

Combinations of hierarchies

Sequential combinations by chaining

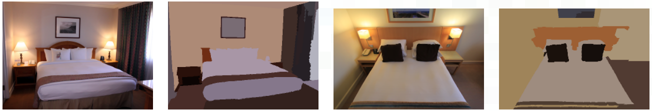
Best segmentation $(\mathcal{H}^*, \lambda^*)$ in sequential combinations for a given score [11].

Simplified Mumford-Shah



(a) (b) (c) (d)

Weighted-Human Disagreement Rate



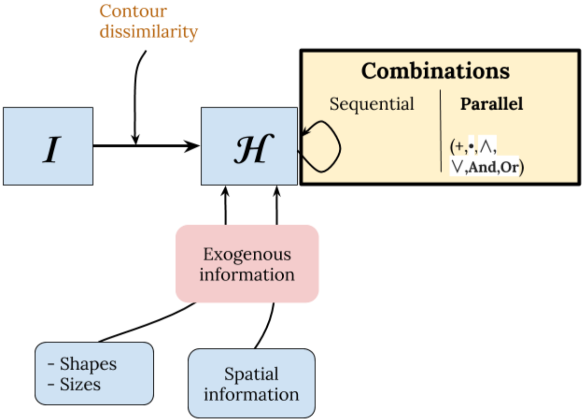
(a) (b) (c) (d)

[11] Fehri, A., Velasco-Forero, S., & Meyer, F. (2016, August). Automatic selection of stochastic watershed hierarchies. In EUSIPCO, 2016 (pp. 1877-1881). IEEE.

Combinations of hierarchies

Hierarchies can be combined to express complex properties

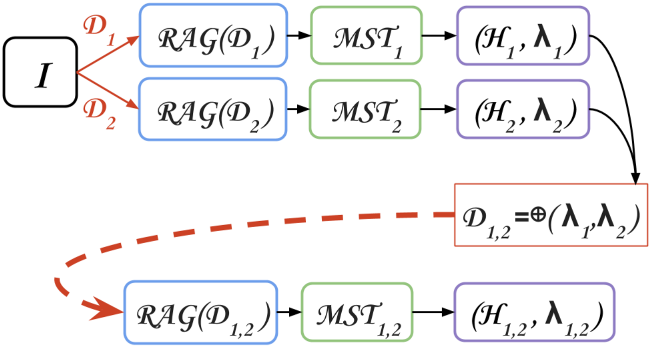
- **Parallel (algebraic) combinations**: supremum, infimum, linear combination, and, or, not



Combinations of hierarchies

Parallel (algebraic) combinations

- General case



Combinations of hierarchies

Parallel (algebraic) combinations

Type of combination	Associated ultrametric
Lattice of hierarchies	
INF $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	SUP$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
SUP $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	INF$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
Probabilistic combinations	
AND $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2$
OR $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 - (\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2)$
NOT $((\mathcal{H}, \boldsymbol{\lambda}))$	$1 - \boldsymbol{\lambda}$
Statistical combinations	
MEAN $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\frac{1}{2}(\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2)$
LC $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\alpha \times \boldsymbol{\lambda}_1 + \beta \times \boldsymbol{\lambda}_2$

Combinations of hierarchies

Parallel (algebraic) combinations

Type of combination	Associated ultrametric
Lattice of hierarchies	
INF $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	SUP$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
SUP $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	INF$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
Probabilistic combinations	
AND $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2$
OR $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 - (\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2)$
NOT $((\mathcal{H}, \boldsymbol{\lambda}))$	$1 - \boldsymbol{\lambda}$
Statistical combinations	
MEAN $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\frac{1}{2}(\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2)$
LC $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\alpha \times \boldsymbol{\lambda}_1 + \beta \times \boldsymbol{\lambda}_2$

- Order relation between hierarchies.
  $\rightarrow$ SUP, INF of two hierarchies
- The supremum of two ultrametrics is an ultrametric.
- In general, other operators do not produce an ultrametric.

Combinations of hierarchies

Parallel (algebraic) combinations

Type of combination	Associated ultrametric
Lattice of hierarchies	
INF $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	SUP$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
SUP $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	INF$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
Probabilistic combinations	
AND $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2$
OR $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 - (\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2)$
NOT $((\mathcal{H}, \boldsymbol{\lambda}))$	$1 - \boldsymbol{\lambda}$
Statistical combinations	
MEAN $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\frac{1}{2}(\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2)$
LC $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\alpha \times \boldsymbol{\lambda}_1 + \beta \times \boldsymbol{\lambda}_2$

- SWS hierarchies $\rightarrow$ ultrametric expressing the probabilities of simple events implying markers.
- Can be combined using boolean logical operators to express more complex events.

Combinations of hierarchies

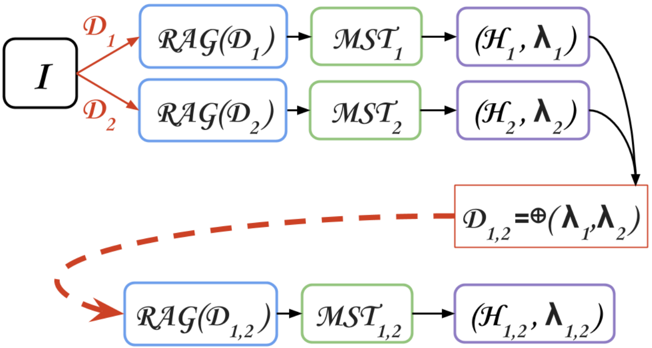
Parallel (algebraic) combinations

Type of combination	Associated ultrametric
Lattice of hierarchies	
INF $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	SUP$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
SUP $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	INF$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$
Probabilistic combinations	
AND $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2$
OR $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 - (\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2)$
NOT $((\mathcal{H}, \boldsymbol{\lambda}))$	$1 - \boldsymbol{\lambda}$
Statistical combinations	
MEAN $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\frac{1}{2}(\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2)$
LC $((\mathcal{H}_1, \boldsymbol{\lambda}_1), (\mathcal{H}_2, \boldsymbol{\lambda}_2))$	$\alpha \times \boldsymbol{\lambda}_1 + \beta \times \boldsymbol{\lambda}_2$

- Any other combination is possible.
- Mean, median, linear combinations.

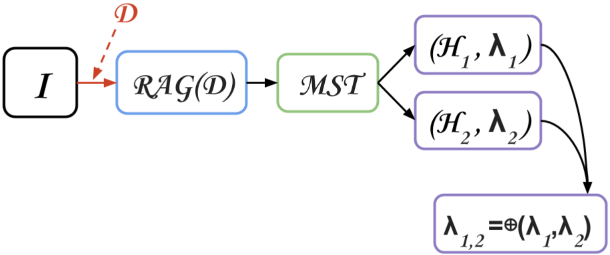
Combinations of hierarchies

Parallel (algebraic) combinations - General case



Combinations of hierarchies

Parallel (algebraic) combinations - Simpler case



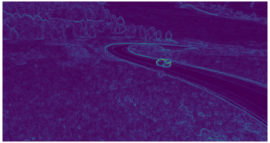
Condition

$\bigoplus$ s.t. $\forall(x_1, x_2, y_1, y_2) \in \mathbb{R}^4_+, (x_1 \leq x_2)$ and $(y_1 \leq y_2) \Rightarrow \bigoplus(x_1, y_1) \leq \bigoplus(x_2, y_2)$

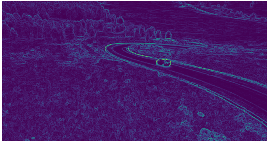
Combinations of hierarchies

Example

(a) Image



(b) LAB gradient



(c) Gradient over green channel

(d) $\mathcal{H}_{vol}^{Gr(RGB)}$: 10 regions

(e) $\mathcal{H}_{vol}^{Gr(RGB)}$: 30 regions

(f) $\mathcal{H}_{vol}^{Gr(RGB)}$: 50 regions

Combinations of hierarchies

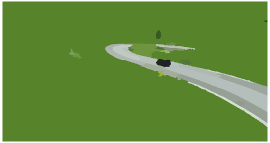
Example

(a) $\mathcal{H}_{surf}^{Gr(RGB)}$: 10 regions

(b) $\mathcal{H}_{surf}^{Gr(RGB)}$: 30 regions

(c) $\mathcal{H}_{surf}^{Gr(RGB)}$: 50 regions

(d) $\mathcal{H}_{trivial}^{Gr(G)}$: 10 regions



(e) $\mathcal{H}_{trivial}^{Gr(G)}$: 30 regions

(f) $\mathcal{H}_{trivial}^{Gr(G)}$: 50 regions

Combinations of hierarchies

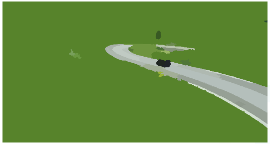
Example

(a) $\mathcal{H}_{surf}^{Gr(RGB)}$: 10 regions

(b) $\mathcal{H}_{surf}^{Gr(RGB)}$: 30 regions

(c) $\mathcal{H}_{surf}^{Gr(RGB)}$: 50 regions

(d) $\mathcal{H}_{trivial}^{Gr(G)}$: 10 regions



(e) $\mathcal{H}_{trivial}^{Gr(G)}$: 30 regions

(f) $\mathcal{H}_{trivial}^{Gr(G)}$: 50 regions

(g) $\mathcal{H}_{\mathrm{INF}}$: 10 regions

(h) $\mathcal{H}_{\mathrm{INF}}$: 30 regions

(i) $\mathcal{H}_{\mathrm{INF}}$: 50 regions

Plan

Introduction

Stochastic Watershed (SWS) Hierarchies

Introducing prior spatial information

Combinations of hierarchies

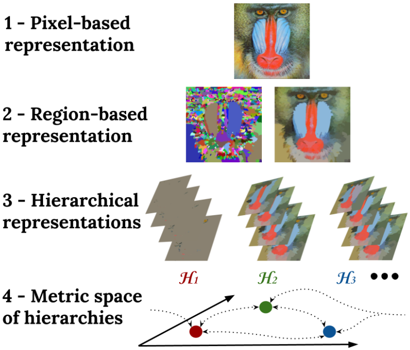
Structuring the space of hierarchies

Conclusion

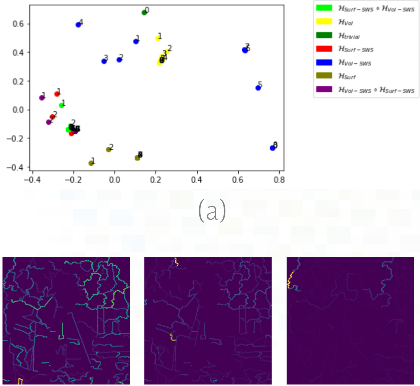
Structuring the space of hierarchies

→ Explosion of the number of possible hierarchies

Gromov-Hausdorff distance between hierarchies



Dimensionality reduction, data analysis

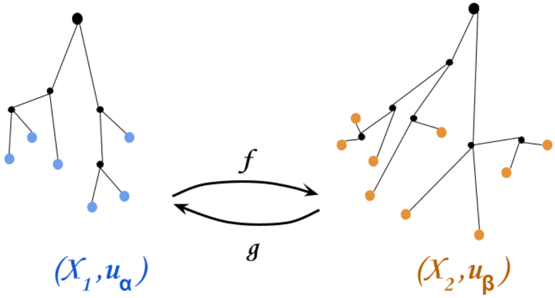


(a)

(b) (c) (d)

Gromov-Hausdorff distance between hierarchies

General case

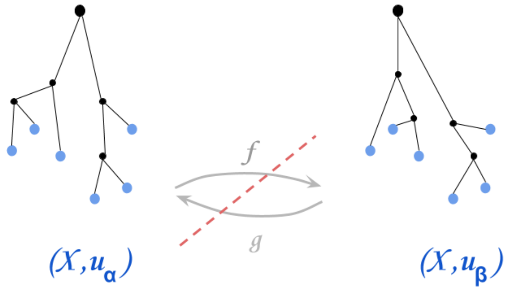


Definition:

- $\mathrm{d}_{\mathrm{GH}}(X_1, X_2) := \frac{1}{2} \min_{f,g} \max(dis(f), dis(g), dis(f,g))$
- $\begin{cases} dis(f) := \max_{(x,x') \in X_1^2} |u_\alpha(x,x') - u_\beta(f(x), f(x'))| \\ dis(f,g) := \max_{x \in X_1, x' \in X_2} |u_\alpha(x, g(x')) - u_\beta(x', f(x))| \end{cases}$

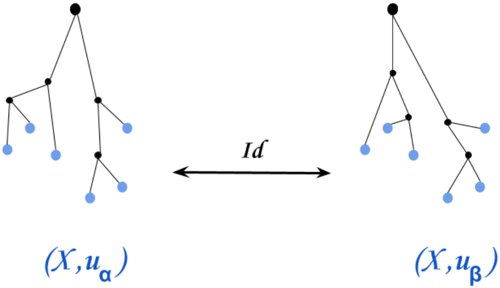
Gromov-Hausdorff distance between hierarchies

Simplest case



Gromov-Hausdorff distance between hierarchies

Simplest case

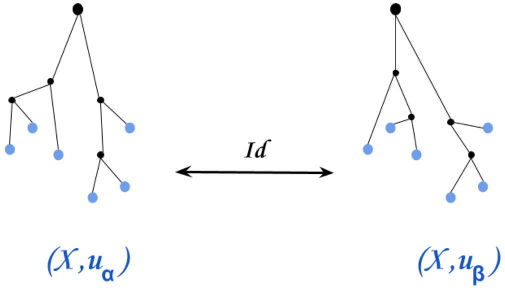


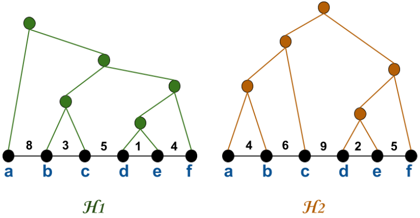
It simply becomes:

- $\mathrm{d}_{\mathrm{GH}}((X, u_\alpha), (X, u_\beta)) = \max_{x,x' \in X} |u_\alpha(x, x') - u_\beta(x, x')|$

Gromov-Hausdorff distance between hierarchies

Simplest case



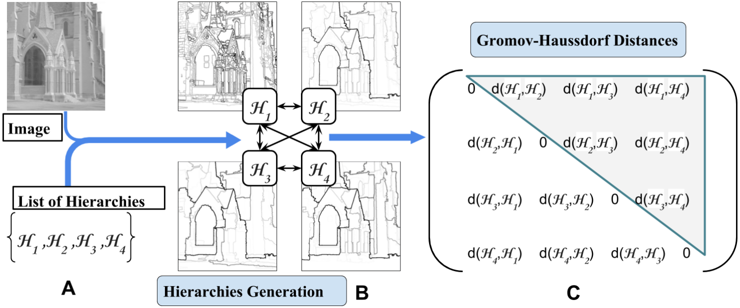


$$d_{\mathcal{GH}}(\mathcal{H}1,\mathcal{H}2) = max\,(\,|8\text{-}4\,|,\,|3\text{-}6\,|,\,|5\text{-}9\,|,\,|1\text{-}2\,|,\,|4\text{-}5\,|) \\ = 4$$

It simply becomes:

- $\mathrm{d}_{\mathrm{GH}}((X,u_\alpha),(X,u_\beta)) = \max_{x,x' \in X} |u_\alpha(x,x') - u_\beta(x,x')|$

Characterizing images by interhierarchy distances



- Multiplying points of views on the same image
- The distances between hierarchies provides valuable information
- **New features**: interhierarchy distances

Experiments and results

Dead-leaves simulated images

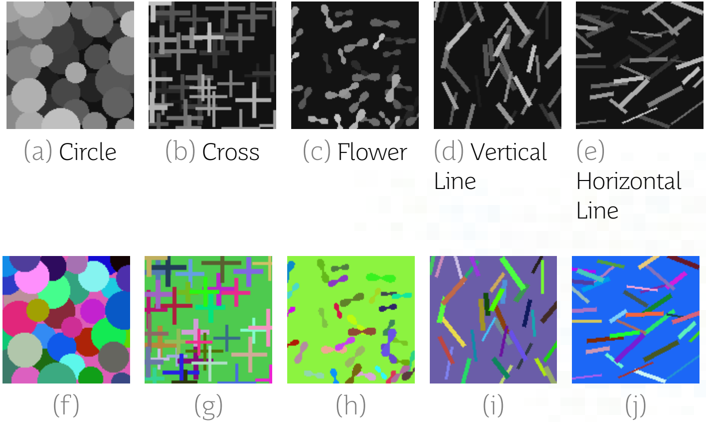
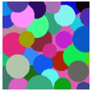
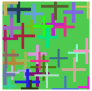


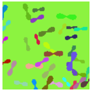
Figure : Simulated images by dead leaves model with different primary grains.

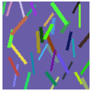
Experiments and results

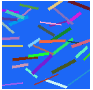
Feature generation

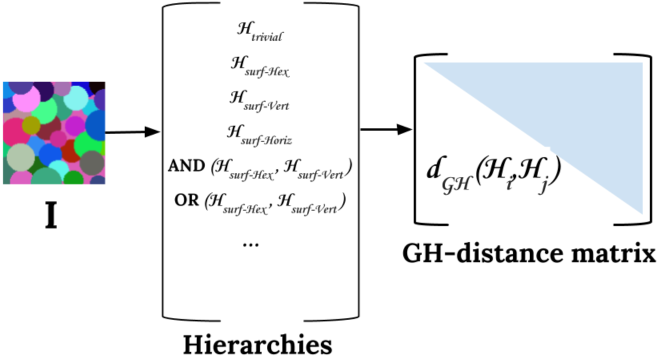

(a)


(b)


(c)

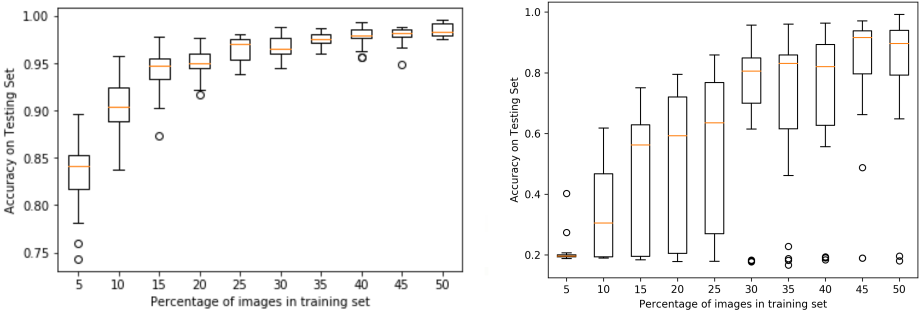

(d)


(e)



Experiments and results

“Aha” moment [12]



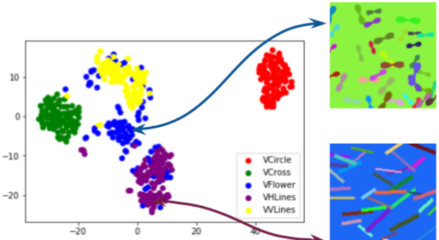
(f) (g)

Figure : Classification error vs the number of images in the training set (25 repetitions) : (a) Linear SVM on proposed features, (b) CNN.

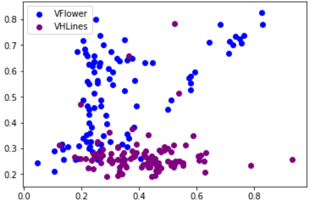
[12] Yan Z, Zhou XS. How intelligent are convolutional neural networks?. arXiv preprint arXiv:1709.06126. 2017 Sep 18.

Experiments and results

Understandability



(a) 2D scatterplot by t-SNE



(b) 2D scatterplot by L_1-SVM

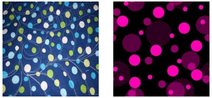
- Most discriminative feature: $d_{GH}(\mathcal{H}_{surf-VertSE}, \mathcal{H}_{\text{AND}(surf-VertSE,surf-HexSE)})$

Experiments and results

Texture classification

(c) Banded

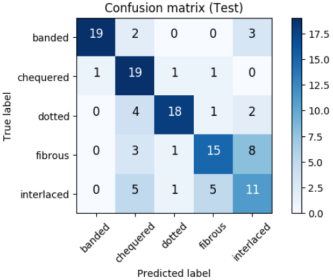
(d) Chequered



(e) Dotted

(f) Fibrous

(g) Interlaced



Plan

Our contributions

- Various morphological hierarchical representations
- Versatile approach to introduce spatial prior information for hierarchical segmentation
- Combination of hierarchies
- Methodology to study the space of hierarchies
- Interhierarchy distance matrices as powerful geometric features
- Hierarchical representations module in the open-source Smil library

Perspectives

- Extension to other types of graphs
- The MST is usually not unique: methods to avoid an arbitrary choice
- Interhierarchy distances matrices for unsupervised image classification
- Local contour descriptors as signatures of saliencies
- Refine the output of a segmentation module by exploring a hierarchy with more details in the zones of interest

Personal publications

- Fehri, A., S. Velasco-Forero, and F. Meyer (2016). « Automatic Selection of Stochastic Watershed Hierarchies ». In: 24th European Signal Processing Conference. IEEE, pp. 1877–1881.
- Fehri, A., S. Velasco-Forero, and F. Meyer (2017). « Prior-based Hierarchical Segmentation Highlighting Structures of Interest ». In: International Symposium on Mathematical Morphology and Its Applications to Signal and Image Processing. Springer, pp. 146–158.
- Fehri, A., S. Velasco-Forero, and F. Meyer (2017). « Segmentation hiérarchique faiblement supervisée ». In: Actes du 26e Colloque GRETSI, Juan-Les-Pins, France.
- Fehri, A., S. Velasco-Forero, and F. Meyer (2018). « Characterizing Images by the Gromov-Hausdorff Distances Between Derived Hierarchies ». In: 2018 IEEE International Conference on Image Processing (ICIP).

Thank you for your attention.