CHARACTERIZING IMAGES BY THE GROMOV-HAUSDORFF DISTANCES BETWEEN DERIVED HIERARCHIES

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Hierarchical segmentation: a scale-space representation

1. Information is intrinsically multi-scale:
   - Single-scale observation is very restrictive
   - Need for multi-scale representations
2. Hierarchies on graphs: Cutting edges of the Minimum Spanning Tree by decreasing valuations → progressive fusion of regions
   - Minimum Spanning Forest (MSF) hierarchy $(\mathcal{H}, \boldsymbol{\lambda})$
   - $\boldsymbol{\lambda}$: **ultrametric distance**
   - Can be modeled as a tree called **dendrogram**
   - Can be visualized as a **saliency map**

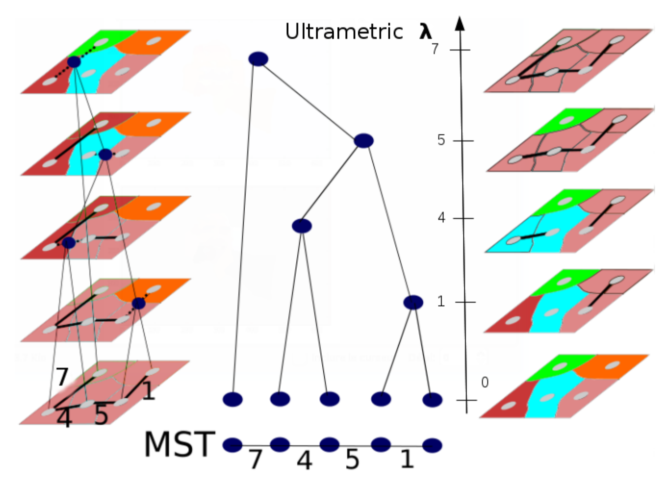


Figure: A hierarchy $(\mathcal{H}, \boldsymbol{\lambda})$

Multiplying the viewpoints on the same image

A single hierarchy is usually not sufficient:

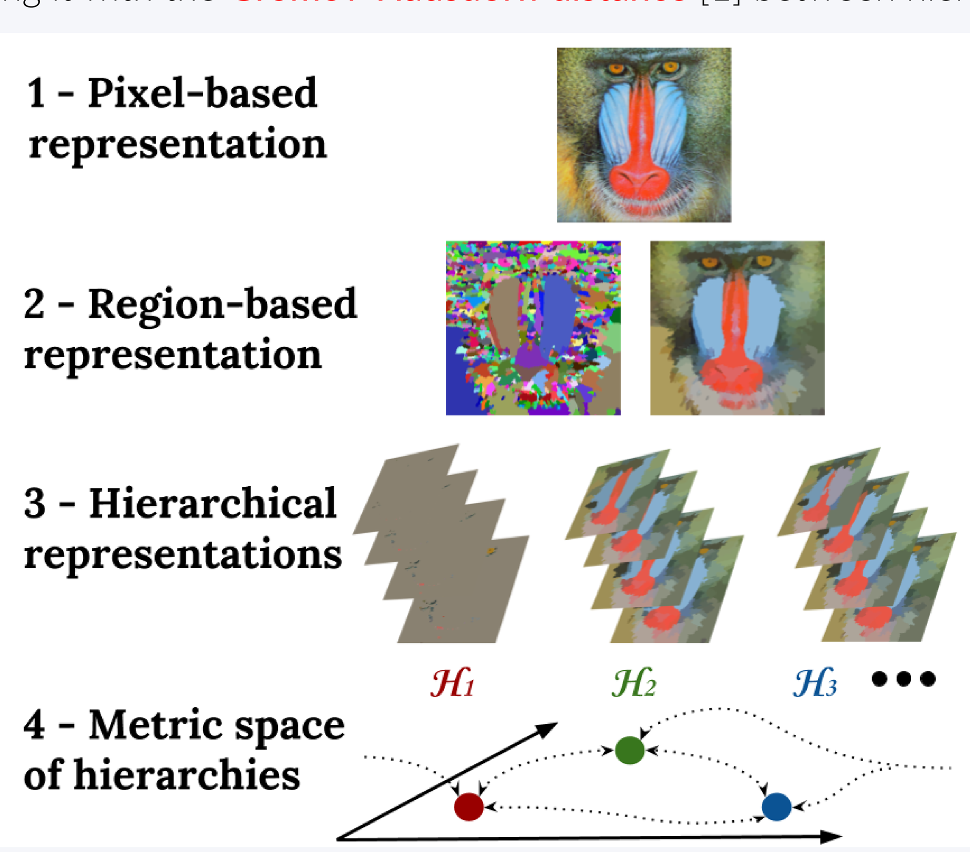
- There is no single hierarchy that captures all the desired features.

→ **Multi-model** approach by considering **several hierarchies**.
→ We work with morphological hierarchies [2], but our approach is extendable to any type of hierarchy.

(a) Image
(b) Trivial hierarchy (c) Area-based SWS hierarchy (d) Volume-based SWS hierarchy

Structuring the space of hierarchies

- We can structure the space of hierarchies itself into a metric space
- We do so by providing it with the Gromov-Hausdorff distance [1] between hierarchies.



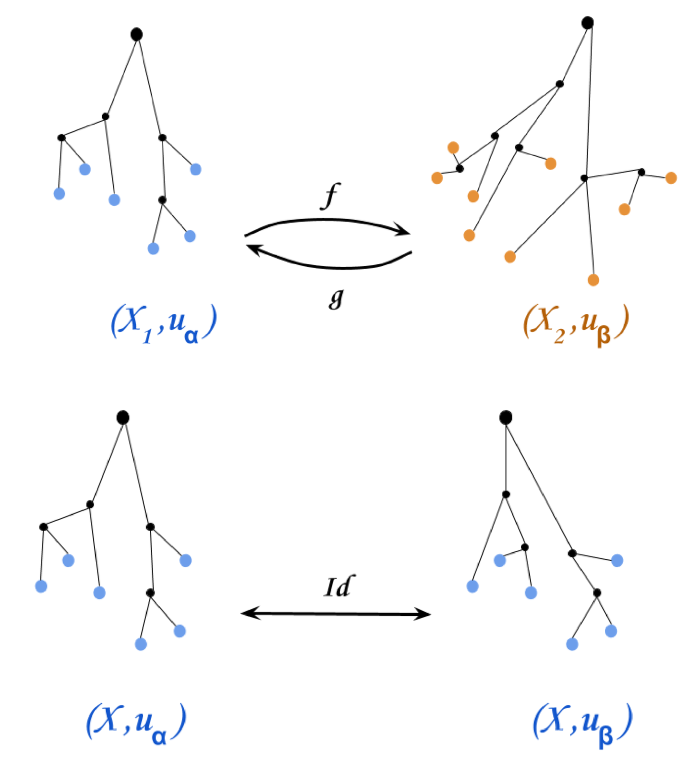
Gromov-Hausdorff distance between hierarchies

Definition [1]:

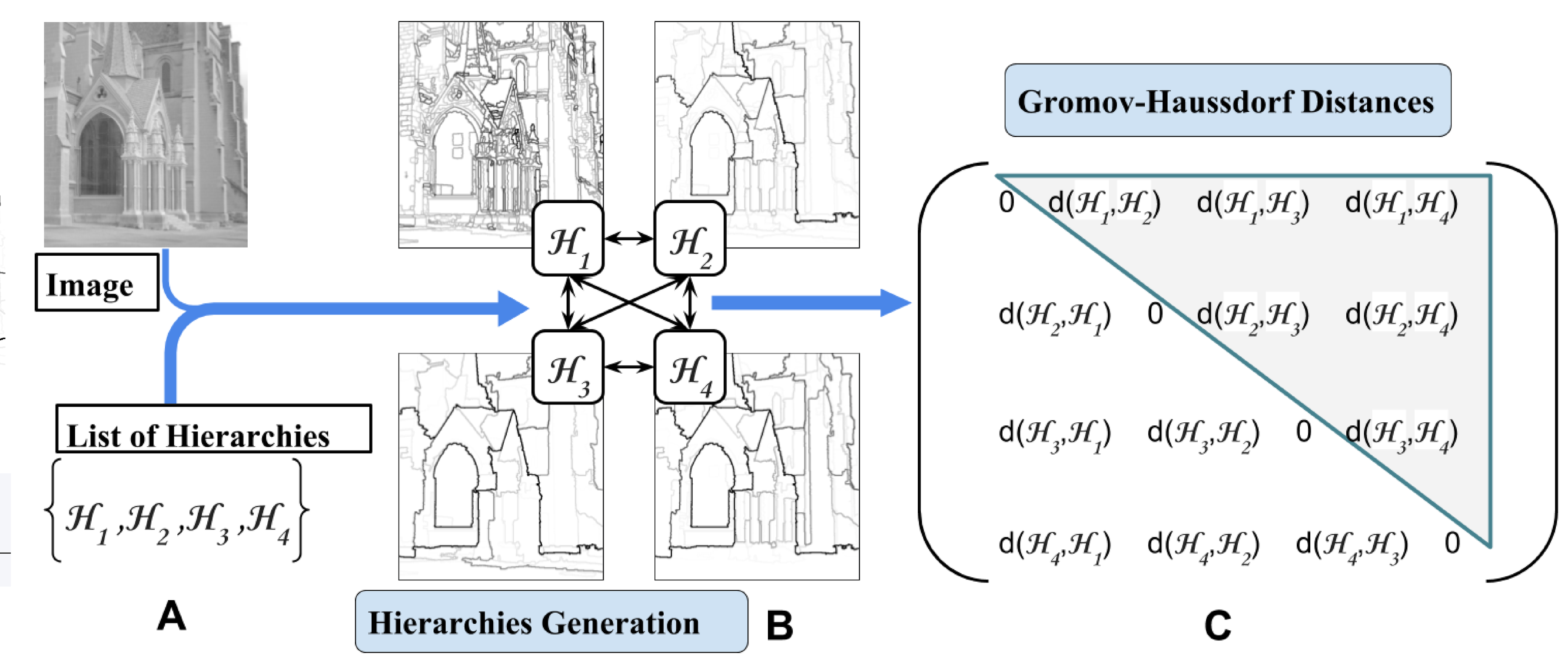
- $d_{GH}((X_1, u_\alpha), (X_2, u_\beta)) := \frac{1}{2}\min_{f,g}\max(dis(f), dis(g), dis(f,g))$
- $\begin{cases} dis(f) := \max_{(x,x')\in X_1^2} |u_\alpha(x,x') - u_\beta(f(x), f(x'))| \\ dis(f,g) := \max_{x\in X_1, x'\in X_2} |u_\alpha(x, g(x')) - u_\beta(x', f(x))| \end{cases}$

In our case, the data points are on the same graph so that it simply becomes:

- $d_{GH}((X, u_\alpha), (X, u_\beta)) = \max_{x,x'\in X} |u_\alpha(x,x') - u_\beta(x,x')|$

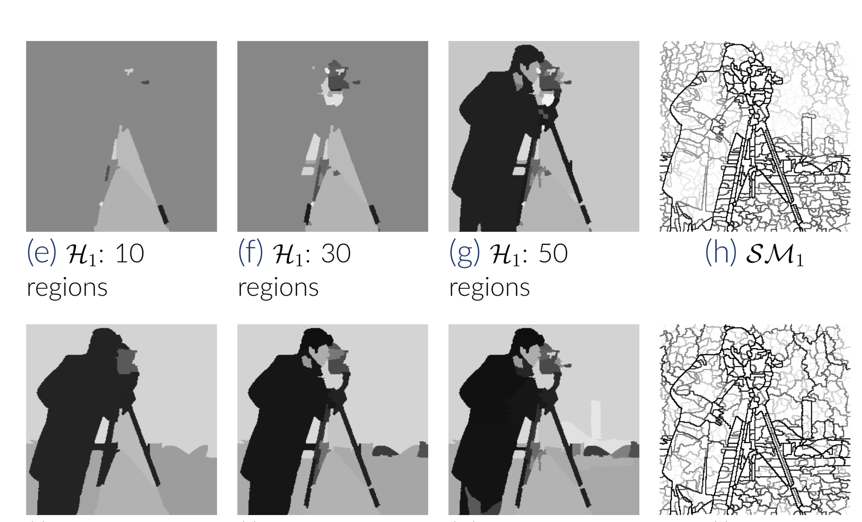


Characterizing images by interhierarchy distances



- Multiplying points of views on the same image
- The distances between hierarchies provides valuable information
- New features: interhierarchy distances

- Example:
  Let us consider two hierarchies:
  - $(\mathcal{H}_1, \boldsymbol{\lambda}_1)$ (highlighting contrasted contours)
  - $(\mathcal{H}_2, \boldsymbol{\lambda}_2)$ (highlighting contrasted contours between big regions)



(e) $\mathcal{H}_1$: 10 regions (f) $\mathcal{H}_1$: 30 regions (g) $\mathcal{H}_1$: 50 regions (h) $\mathcal{SM}_1$
(i) $\mathcal{H}_2$: 10 regions (j) $\mathcal{H}_2$: 30 regions (k) $\mathcal{H}_2$: 50 regions (l) $\mathcal{SM}_2$

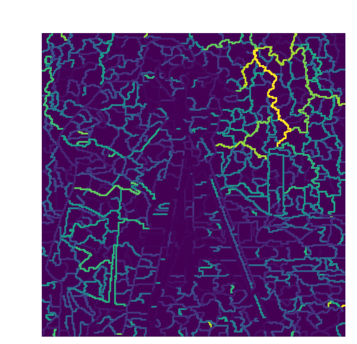
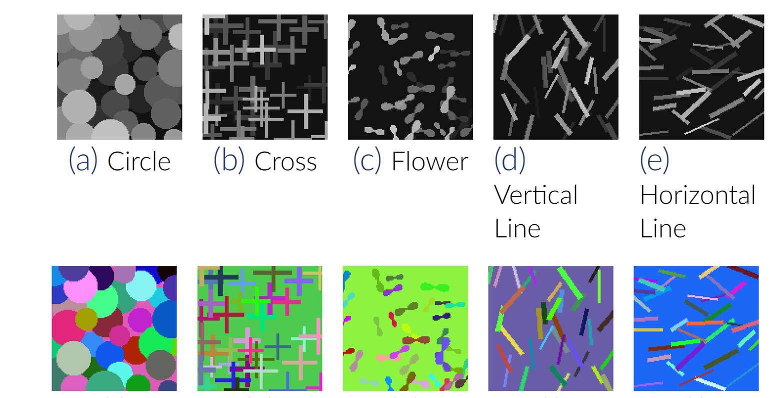


Figure: Difference of saliency maps $|\mathcal{SM}_2 - \mathcal{SM}_1|$.

We have: $d_{GH}(\mathcal{H}_1, \mathcal{H}_2) = \max(|\mathcal{SM}_2 - \mathcal{SM}_1|)$.

Experiments and results

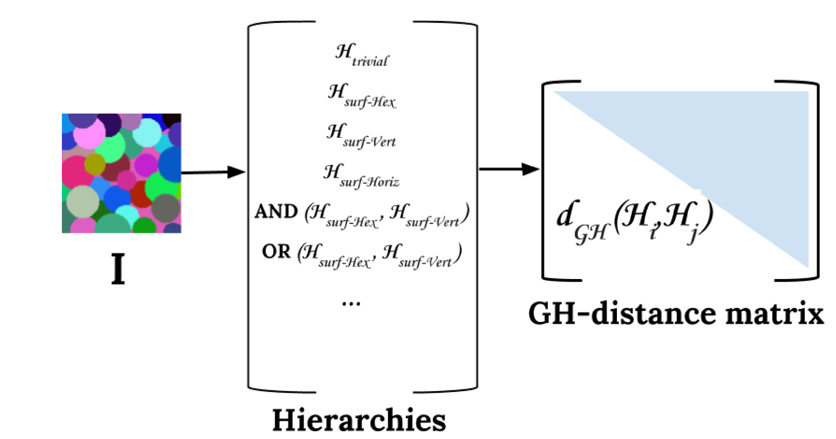
Dead-leaves simulated images



(a) Circle (b) Cross (c) Flower (d) Vertical Line (e) Horizontal Line
(f) (g) (h) (i) (j)

Figure: Simulated images by dead leaves model with different primary grains.

Feature generation



Results

- Learning efficiency: (``Aha'' moment [3] of sudden clarity) - Our features vs CNN

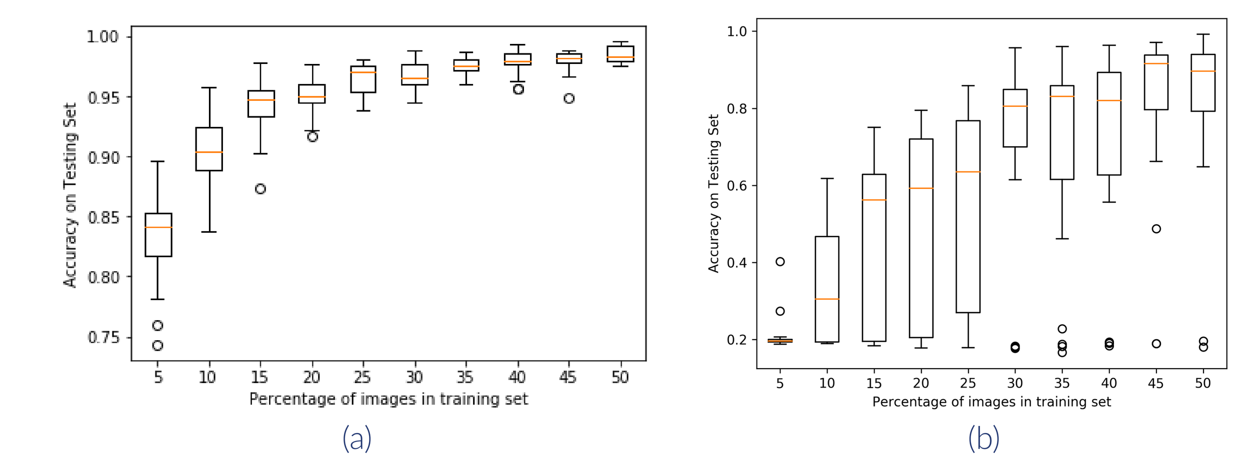
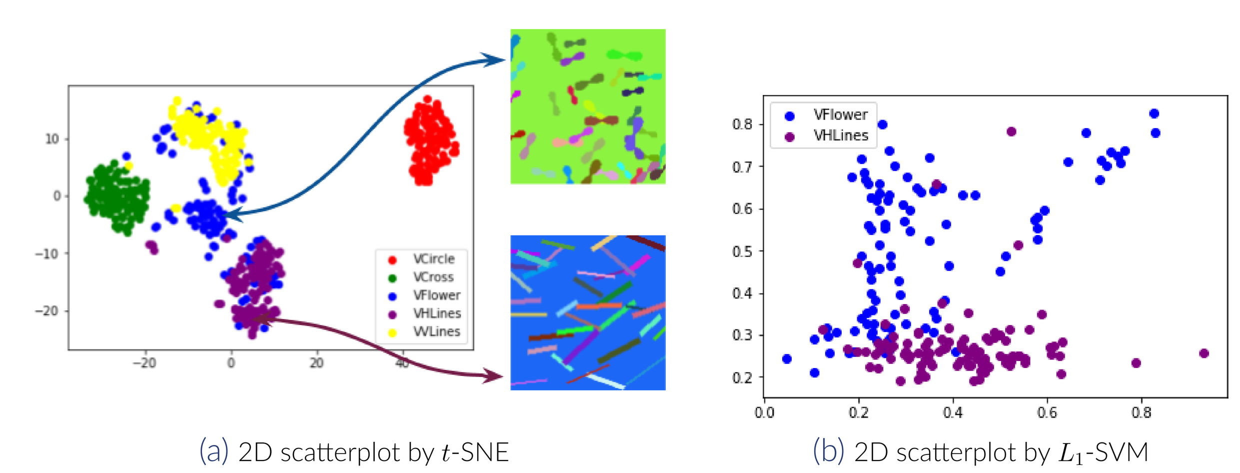


Figure: Classification error vs the number of images in the training set (25 repetitions) : (a) Linear SVM on proposed features, (b) CNN.

- Understandability



(a) 2D scatterplot by t-SNE (b) 2D scatterplot by L_1-SVM

Figure: Discriminating between the classes "Flowers" and "Horizontal Lines" is not straightforward, but it is made much easier using our features. Using the variable selection method $L1$-SVM, we can isolate the more discriminative distances for two specific classes to separate. In this case, the most discriminative feature is: $d_{GH}(\mathcal{H}_{surf-VertSE}, \mathcal{H}_{\text{AND}(surf-VertSE, surf-HexSE)})$. It provides a geometrical interpretation of the image.

References

[1] M. Gromov.
Metric structures for Riemannian and non-Riemannian spaces.
Springer Science & Business Media, 2007.

[2] F. Meyer.
Stochastic watershed hierarchies.
In *Advances in Pattern Recognition (ICAPR), 2015 Eighth International Conference on*, pages 1--8. IEEE, 2015.

[3] Z. Yan and X. S. Zhou.
How intelligent are convolutional neural networks?
arXiv preprint arXiv:1709.06126, 2017.